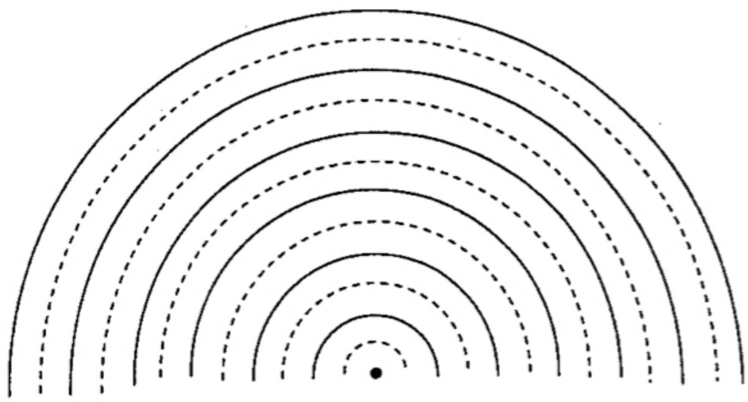
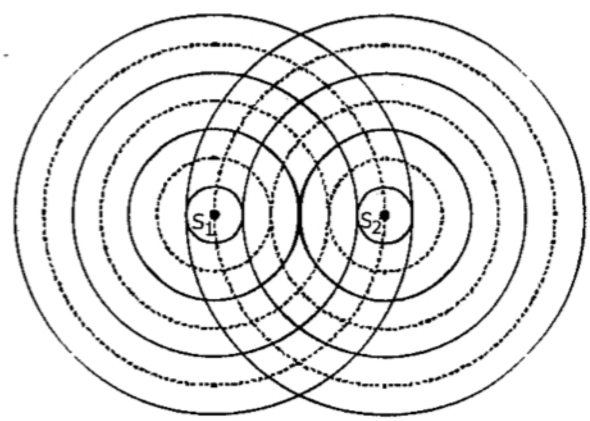
Interference in Two-Dimensions

Waves in two dimensions show constructive and destructive interference, just like waves in one dimension. In this course, we will limit our study of two-dimensional interference to a specific case: two point sources creating waves of the same frequency (and wavelength) in a uniform medium with no barriers, resulting in a **two-dimensional standing wave**.

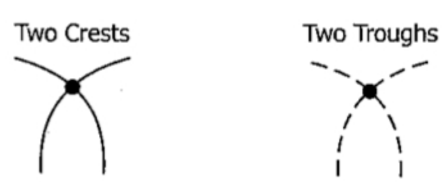
In the diagram below, the circular waves created by a point source are shown. The crests are shown as solid lines and the troughs as dashed lines.



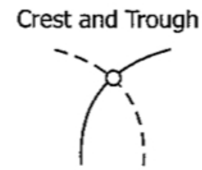
When there are two point sources placed side by side, the circular wave patterns will overlap, as shown below.



Constructive interference will occur at points where either two crests (two solid lines) or two troughs (two dashed lines) cross. Areas of constructive interference will be indicated by a small solid circle, as shown below.



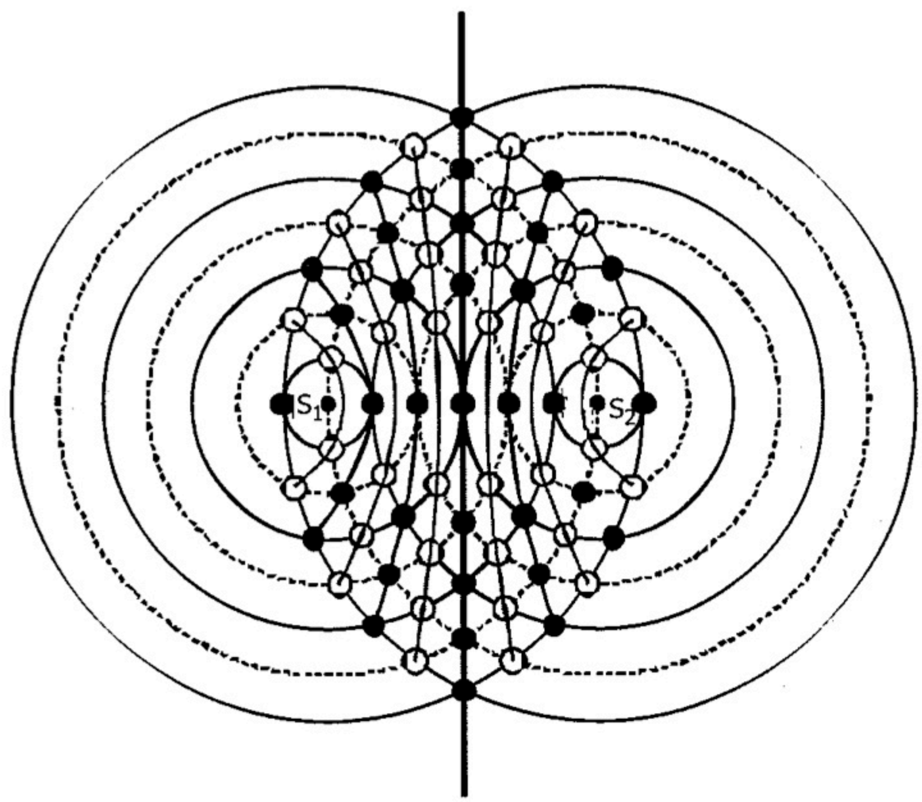
Destructive interference will occur at points where a crest (solid line) and trough (dashed line) cross. Areas of destructive interference will be indicated by a small open circle, as shown below.



The pattern formed by these points of constructive and destructive interference is that of a two-dimensional standing wave. The points of destructive interference are the **nodes** and the points of constructive interference are the **antinodes**.

In a one-dimensional standing wave, the nodes were points along the medium where destructive interference occurred, and antinodes were points where constructive interference occurred. This remains true for two-dimensional standing waves, with one major difference: nodes and antinodes are no longer merely points, but become extended into what we call **nodal lines** and **antinodal lines**.

The diagram below shows all of the nodes and antinodes produced so far. The lines are drawn to illustrate the position of the nodal and antinodal lines in the pattern.

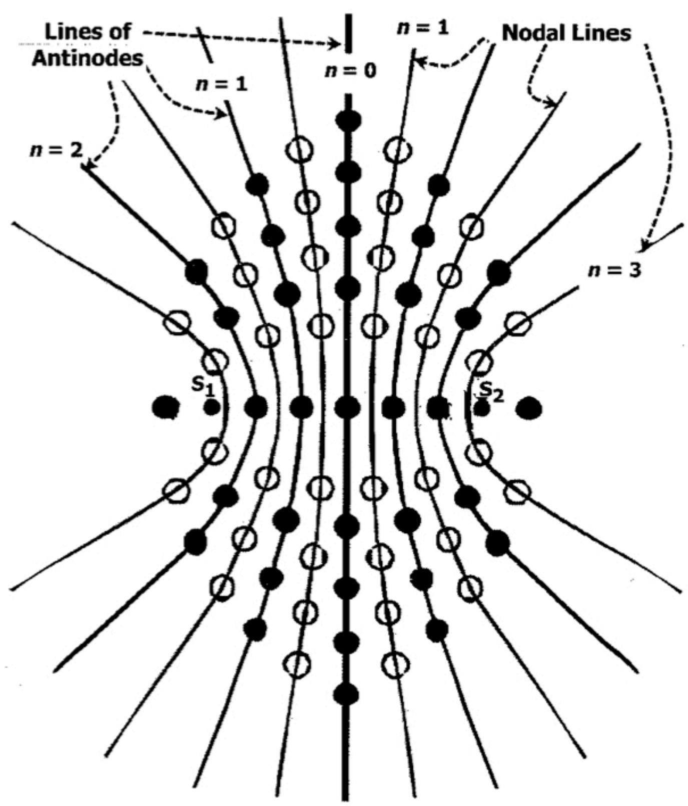


To get a better picture of what the interference pattern looks like, visit the following website:

http://www.falstad.com/ripple/

Change the top menu to **Setup: Double Slit** and you will be able to clearly view the standing wave pattern, plus the nodal lines.

The pattern of nodal and antinodal lines is called an interference pattern. The nodal lines are numbered in each quadrant as shown in the diagram below.



The interference pattern will remain stationary as long as three factors do not change:

1. The frequency of the sources.
    - increasing frequency will decrease wavelength, resulting in more nodal lines
    - decreasing frequency will increase wavelength, resulting in less nodal lines

2. The distance between the sources.
    - increasing the distance between the sources will increase the number of nodal lines
    - decreasing the distance between the sources will decrease the number of nodal lines

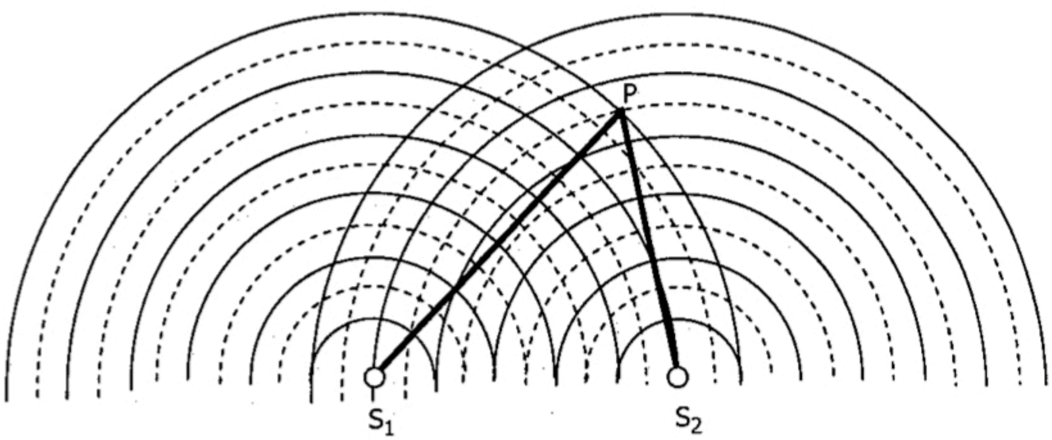
3. The relative phase of the sources.
    - all waves we consider will be in phase, so this is not a consideration for us

Path Length Difference

The path length difference between two point sources, S_1 and S_2, and a point P on a nodal line is given by the absolute value of the difference in length between the path from S_1 to P and S_2 to P.

$$PLD = |PS_1 - PS_2|$$

Example 1
In the diagram below, the distance from S_1 to P is $4.8\ cm$ and the distance from S_2 to P is $3.6\ cm$. What is the path length difference?



In the diagram above, the point P is located on the second nodal line. It is worth noting that the path lengths PS_1 and PS_2 can also be counted in wavelengths: $PS_1 = 6\lambda$ and $PS_2 = 4.5\lambda$, giving a PLD of 1.5λ. It turns out that there is a simple relationship between the number of the nodal line that the point P is on and the PLD.

PLD and Nodal Lines

The path length difference to a point P_n on a given nodal line, n, is given by

$$PLD = \left|PS_1 - PS_2\right| = \left(n - \frac{1}{2}\right)\lambda$$

Example 2

In an interference pattern for two point sources, the path length from source 1 to a point P_n is $12.27\ cm$, and the path length from source 2 to the same point is $14.74\ cm$. The point is on the fourth nodal line.

a) What is the path length difference to P_n?

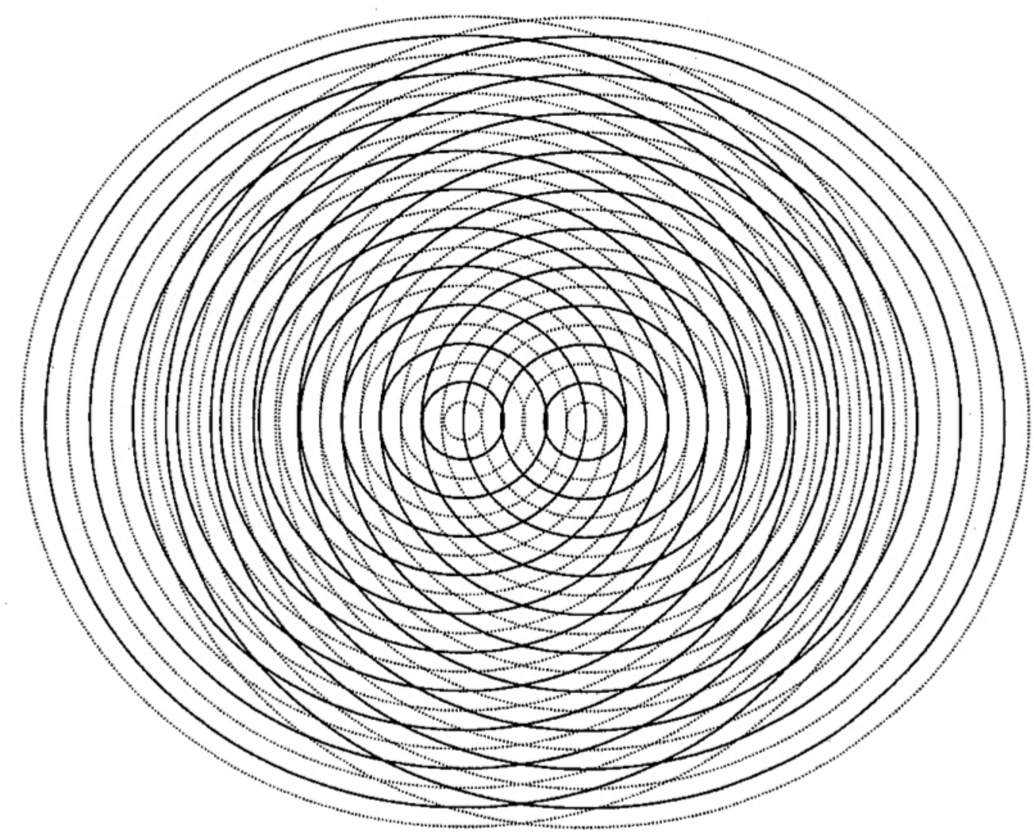
b) What is the wavelength of these waves?

Waves Worksheet #8

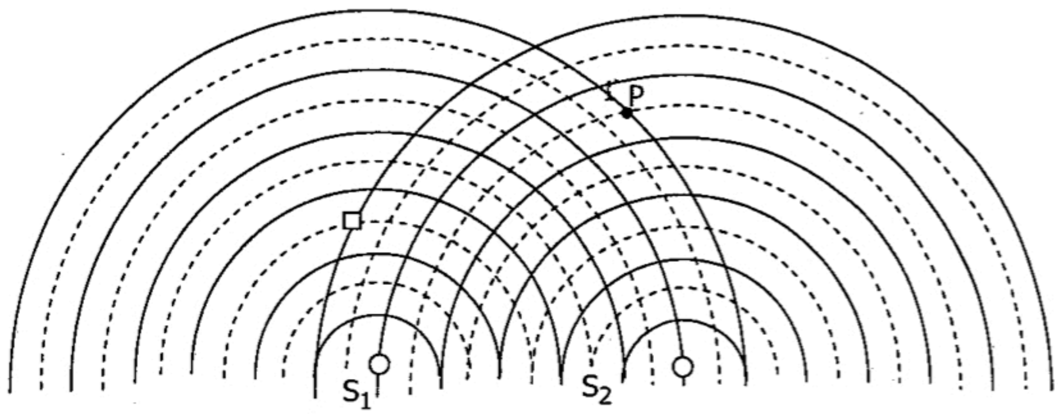
1. When the waves from two point sources meet on the surface of the water, what must there be to produce

    a) constructive interference?
    b) destructive interference?
    c) an antinode?
    d) a node?

2. On the pattern for two point sources given below, label source 1 as S_1 on the left side of the pattern and label source 2 as S_2 on the right side of the pattern. Draw in a horizontal line through the two sources. Find the midpoint of the line segment S_1S_2 and draw in the perpendicular bisector of this line segment. This is called the center line. Mark in all of the nodes with an open circle and all of the antinodes with a shaded circle.



3. Two point sources are generating waves at the same frequency and phase. The pattern is shown below. The solid lines represent the crests and the dashed lines represent the troughs.



Draw in the perpendicular bisector of S_1S_2.

a) Draw the **second nodal line to the left** and the **fourth nodal line to the right** of the perpendicular bisector.
b) What kind of interference, constructive or destructive, is occurring at the location of the square?
c) On which nodal line would the square be located?
d) What would be the distance between the square and the source S_2 if the wavelength of the waves is $4.0\ cm$?
e) On which nodal line is point P? What is the path length difference to point P?
f) What is the path length difference to the square?

4. Two point sources are generating waves in a ripple tank, causing the waves to interfere. The two point sources are $10.0\ cm$ apart, and the frequency of the waves is $4.0\ Hz$. A point on the first nodal line is located $16.0\ cm$ away from one source and $15.0\ cm$ away from the other.

a) What is the wavelength of the waves?
b) What is the speed of the waves?

5. State the effect on the interference pattern for two point sources if

a) the wavelength of the waves was increased.
b) the two sources were moved closer together.
c) the frequency of the waves was increased.